Mathematics: The Language of Science

Overview

Description

This activity will show students the types of mathematics that show up very frequently in college science classes. They will be asked to solve for variables involving systems of equations as well as manipulate exponential and logarithmic expressions. The students will also be asked to show mastery of graphing/sketching skills involving equations that contain variables and constants but not numbers. Finally, students will be asked to show skill in converting nonlinear mathematical equations into linear graphs.

Final Product: Students will create a packet of full-page plots of various functions that have been rearranged to be plotted in a linear manner. Additionally, if required by the instructor, students will provide a written explanation for why such linear transformations are useful in science and include some examples that they have found within a specific discipline.

Subject

Chemistry, Physics, Algebra II, Pre-Calculus

Note: Portions of this CRA can be completed with Algebra II-level math, and some portions require Trig/Pre-Cal knowledge.

Task Level

Grade 10-12

Objectives

Students will:

• Simplify common mathematical functions seen in science classes.
• Rearrange common mathematical functions.
• Solve simple systems of equations involving multiple equations with multiple variables.
• Create simple sketches of common graphs used in science classes.
• Be able to convey the main components of the functions.
• Plot simple to moderately complicated functions in a linear fashion.
Preparation

- For each student, make a copy of the Student Notes page.
- Provide graph paper for students.

Prior Knowledge

Students should be familiar with the concepts of high school algebra, such as basic functions, systems of equations and graphing, along with basic high school trigonometry, including slopes of lines and rates of change.

Key Concepts and Terms

- Logarithms and exponents
- Sketching vs. graphing
- Slope and intercepts
- Systems of equations

Time Frame

This assignment will require one or two class periods for a total time of approximately two hours. If required, the written activity may require an additional class period, though it may also be completed outside of class.

The assignment can be modified to meet the needs of different classrooms and student ability.
Instructional Plan

Getting Started

Learning Objectives

Students will:

• Review the basic algebraic functions and skills needed in science classes.

Procedure

1. Individually, and without the use of technology, have the students complete the warm-up activities found in Activity 1.

2. In Activity 1a have the students solve for the variables “A” and “B,” assuming that the other variables are non-zero constants (solving systems of equations for multiple unknowns).

3. In Activity 1b have the students reduce the equations to the simplest form.

4. In Activity 1c have the students solve for variable “B,” while assuming all the others are non-zero constants.

Investigating

Learning Objectives

Students will:

• Review the basic sketching of common science functions.

• Understand the importance of quickly and accurately determining the correct trends and intended purposes of plots.

Procedure

1. Have the students work alone and sketch the plots for the equations found in Activity 2a.

2. Remind them that sketching is not the same as plotting and that the main idea of sketching is to capture the essence of the equation in graphical form. Also remind them that the sketches should label any characteristic values (e.g., y-intercepts, slopes, etc.).

3. When they complete their graphs for Activity 2a, have them get together in pairs to compare and correct their sketches, making sure they discuss any differences. They should re-sketch their graphs on the full page graph paper provided which will be turned in at the end of the assignment.

4. Have the students follow the same set of procedures for Activity 2b.

5. Have the students follow the same set of procedures for Activity 2c.
Drawing Conclusions

**Learning Objectives**

Students will:

- Be able to create linear graphical sketches of simple nonlinear equations.
- (if written component is included) Be able to describe how such mathematical transformations are used in the sciences.

**Procedure**

1. Remind students how to change variables (e.g., for the equation, \( y = x^2 \) we can rewrite as \( y = z \) if \( z = x^2 \)).

2. Have the students work in pairs to come up with sketches of the equations in Activity 3 that they can turn in. The sketches should be straight lines and should also be labeled with what the slope, y-intercept, x-variable and y-variables are for that particular equation.

3. If written component is included, have students individually describe at least one example of how such mathematical transformations are used in a particular branch of science. For examples, please see the Scaffolding section.
**Scaffolding/Instructional Support**

The goal of scaffolding is to provide support to encourage student success, independence, and self-management. Instructors can use these suggestions, in part or all together, to meet diverse student needs. The more skilled the student, however, the less scaffolding that he or she will need. Some examples of scaffolding that could apply to this assignment include:

- Remind students of what a normal straight line looks like in both equation and sketch.
- Remind students that to find the y-intercepts, one can evaluate y when x is equal to zero.
- Remind students that there are four ways to solve systems of equations (combination, substitution, graphical, matrices and determinants). It may be helpful to review some of these methods, or one in particular (substitution, for example, might be particularly relevant and helpful in this activity).
- Remind students that exponents and logarithms are related.
- Remind students that we can create new variables that are related to other variables (e.g., for the equation, \( y=x^2 \) we can rewrite as \( y=z \) if \( z=x^2 \)).
- If an additional written component is included, it might be helpful to share with students some examples of cases in which linear transformations are helpful to scientists. For example, the relationship between brain mass and body mass is roughly linear when both variables are transformed logarithmically (or, when both variables are plotted on a logarithmic scale). Thus, if one wonders whether a given organism has an atypically large or small brain, it is much simpler to visualize and perform statistical analyses if the variables are transformed linearly. Or, as a different example, physicists do not often care about the absolute amount of potential energy but rather care only about the relative amount. Therefore, the addition or removal of a constant term can be a helpful mathematical tool and does not change the physics at all.
Solutions

The information below is intended to help you assess students’ final work products. It may not represent all possible strategies and ideas. The accompanying scoring guide provides specific examples of ways a student might demonstrate content understanding and mastery of cross-disciplinary skills.

**Necessary Elements**

- See the following pages for the attached solutions.
- There may be more solutions that are acceptable, and the ones below are just guidance for what should be included.

**Key Connections**

- Students should remember the basic properties of algebraic manipulation.
- Students should remember the key points of graphing and shifting graphs horizontally and vertically.
- Students should remember the basic properties of logarithms and exponents.
- Students should learn that they can do a change of variable and then plot the new variable to create new graphs with different lines that still contain the same information.

**Common Misconceptions**

- Students may sometimes confuse the algebraic manipulations that result in horizontal shifts in graphs with those that result in vertical shifts in graphs.
- Students may confuse the proper order of operations.
- The concept of change of variable and the linear plotting of nonlinear graphs can be difficult to grasp.
Activity 1a Solutions:

Set 1:

\[ A = 0 \]
\[ B = -d \]

Set 2:

\[ A = d \text{ and } B = 0 \]

and

\[ A = -\frac{1}{3}d \text{ and } B = \frac{2}{3}d \]

Activity 1b Solutions:

\[ e^{a+b+c} \]
\[ e^{a-b-c} \]
\[ e^{-(a+b+c)} \]
\[ \log(xy^4) \]
\[ \ln\left(\frac{1}{x^3}\right) \]
\[ \log\left(\frac{xy^3}{4}\right) \]

Activity 1c Solutions:

\[ B = 10^{4x} \]
\[ B = \frac{1}{2} \times 10^{6x} \]
\[ B = e^{3x} \]
\[ B = 5e^{5x} \]
Activity 2 Solutions:

\[ y = D x + F \]
\[ y = -D x + F \]
\[ y = x^2 \]
\[ y = x^2 + b \]
\[ y = -(x - c)^2 + b \]
\[ y = A \sin(x) \]
\[ y = A \sin(x) + b \]
\[ y = A \sin(x + c) + b \]
\[ y = A \cos(x) \]

\[ y = A \cos(x) + b \]

\[ y = A \cos(x + c) + b \]

\[ y = \frac{1}{x^2} \]

\[ y = \frac{1}{x^2} \]

\[ y = l_o e^x \]

\[ y = l_o e^{-x} \]
Activity 3 Solutions:

\[ h = g^2 + b \]

- x-axis: \( x = g^2 \)
- y-axis: \( y = h \)
- slope: \( m = 1 \)
- y-int: \( B = b \)

\[ \frac{1}{h} = \frac{1}{2} (fg)^2 + \frac{b}{c} \]

- x-axis: \( x = g^{2/3} \)
- y-axis: \( y = 1/h \)
- slope: \( m = (1/2)f^{2/3} \)
- y-int: \( B = b/c \)

\[ h = \frac{1}{g^2} + bc^2 \]

- x-axis: \( x = 1/g^2 \)
- y-axis: \( y = h \)
- slope: \( m = 1 \)
- y-int: \( B = bc^2 \)

\[ f = 2h \left( g^2 + b \right) \]

- x-axis: \( x = g^2 \)
- y-axis: \( y = 1/h \)
- slope: \( m = 2/f \)
- y-int: \( B = 2b/f \)

\[ h = fg^3 + bc^2 \]

- x-axis: \( x = g^3 \)
- y-axis: \( y = h \)
- slope: \( m = f \)
- y-int: \( B = bc^2 \)
### TCCRS Cross-Disciplinary Standards Addressed

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<td><strong>I. Key Cognitive Skills</strong></td>
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<td>A.1. Engage in scholarly inquiry and dialogue.</td>
<td></td>
<td>✓</td>
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<td>A.2. Accept constructive criticism and revise personal views when valid evidence warrants.</td>
<td></td>
<td>✓</td>
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<td>B.1. Consider arguments and conclusions of self and others.</td>
<td></td>
<td>✓</td>
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<td>B.2. Construct well-reasoned arguments to explain phenomena, validate conjectures, or support positions.</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>C.1. Analyze a situation to identify a problem to be solved.</td>
<td>✓</td>
<td>✓</td>
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<td>D.1. Self-monitor learning needs and seek assistance when needed.</td>
<td>✓</td>
<td>✓</td>
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<td>✓</td>
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<td>E.1. Work independently.</td>
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<td>E.2. Work collaboratively.</td>
<td></td>
<td>✓</td>
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<td><strong>II. Foundational Skills</strong> (Note that this applies ONLY if writing activity is included.)</td>
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<tr>
<td>B.1. Write clearly and coherently using standard writing conventions.</td>
<td></td>
<td></td>
<td>✓</td>
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### TCCRS Science and Mathematics Standards Addressed

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<td><strong>II. Algebraic Reasoning (Mathematics)</strong></td>
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<tr>
<td>C.1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to solve equations, inequalities, and systems of linear equations.</td>
<td>✓</td>
<td>✓</td>
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<td><strong>VII. Functions (Mathematics)</strong></td>
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<tr>
<td>B.1. Understand and analyze features of a function.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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</table>
B.2. Algebraically construct and analyze new functions.  

IX. Communication and Representation (Mathematics)  

C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and words.  

X. Connections (Mathematics -- Note that these apply ONLY if writing activity is included.)  

A.2. Connect mathematics to the study of other disciplines.  

I. Nature of Science: Scientific Ways of Learning and Thinking  

C.1. Collaborate on joint projects  

II. Foundation Skills: Scientific Applications of Mathematics  

A.2. Use exponents and scientific notation.  

A.5. Simplify algebraic expressions.  

B.1. Carry out formal operations using standard algebraic symbols and formulae.  

C.2. Understand that a curve drawn on a defined set of axes is fully equivalent to a set of algebraic equations.  

C.3. Understand basic trigonometric principles, including definitions of terms such as sine, cosine, tangent, cotangent, and their relationship to triangles.  

F.3. Understand and use logarithmic notation (base 10).  

III. Foundation Skills: Scientific Applications of Communication (Note that these apply ONLY if writing activity is included.)  

A.1. Use correct applications of writing practices in scientific communication.  

**TEKS Standards Addressed**  

*Mathematics: The Language of Science - Texas Essential Knowledge and Skills (TEKS): Chemistry, Physics and Biology*  

111.33.b.4. Algebra and geometry. The student connects algebraic and geometric representations of functions. The student is expected to:  

111.33.b.8.A. identify and sketch graphs of parent functions, including linear, quadratic, exponential, and logarithmic functions, absolute value of x, square root of x, and reciprocal of x (f(x) = 1/x).
112.39.c.2. (analogous skills are identified in 112.32.c.2.J, 112.33.c.2.G and 112.33.c.2.H, 112.34.c.2.G and 112.34.c.2.H, 112.35.c.2.H and 112.35.c.2.I, 112.36.c.2.G through 112.36.c.2.I, 112.37.c.2.I and 112.37.c.2.K, and 112.38.c.2.D) Scientific processes. The student uses a systematic approach to answer scientific laboratory and field investigative questions. The student is expected to:
- 112.39.c.2.J. organize and evaluate data and make inferences from data, including the use of tables, charts, and graphs;
- 112.39.c.2.K. communicate valid conclusions supported by the data through various methods such as lab reports, labeled drawings, graphic organizers, journals, summaries, oral reports, and technology-based reports; and
- 112.39.c.2.L. express and manipulate relationships among physical variables quantitatively, including the use of graphs, charts, and equations.

112.39.c.6. Science concepts. The student knows that changes occur within a physical system and applies the laws of conservation of energy and momentum. The student is expected to:
- 112.39.c.6.D. demonstrate and apply the laws of conservation of energy and conservation of momentum in one dimension.
Mathematics: The Language of Science

Introduction

In this activity, you will be looking at some of the basic mathematical equations and skills that will be useful for any college-level science course.

Directions

Getting Started

1. On your own and with the use of neither a calculator nor a computer, solve the system of two equations with the two unknowns given in Activity 1a. The unknown variables are “A” and “B,” and you can assume all the other values are constants.

2. Again on your own and without the use of technology, simplify the equations given in Activity 1b.

3. In a similar manner, solve for the variable “B” in the equations given in Activity 1c.

Investigating

1. Listen as your instructor reviews the basics of sketching graphs.

2. On your own, sketch the equations given in Activity 2a. You should include any characteristic values such as y-intercepts or slopes of lines where appropriate. Assume that x and y are the independent and dependent variables, respectively, and the other variables are constant positive values.

3. When you complete your sketches, join up with a partner and compare your graphs to theirs. If there are any variations in the sketches, discuss why you believe yours is right and try to reach a common consensus.

4. When you reach the same conclusion, together make a nice version of the sketch, applying the appropriate labels, to be turned in.

5. Repeat steps 2-4 with the equations found in Activity 2b.

6. Repeat steps 2-4 with the equations found in Activity 2c.

Drawing Conclusions

1. In this section you are going to be sketching slightly more complicated functions but you will transform the sketches to be linear. In pairs, work on plotting the equations in Activity 3 as linear functions where h is the original
dependent variable and $g$ is the original independent variable and all the other variables are constant values. You will need to use the concept of change of variable to accomplish this.

2. As you work on sketching these equations, you will need to show what the new $x$-axis variable, $y$-axis variable, slope and $y$-intercept are for each of the sketches.

3. When you reach a consensus in your group, together make a nice version of the sketch, applying the appropriate labels, to be turned in.

4. If your instructor asks, individually write a brief description (less than 500 words) of an example from a specific branch of science where mathematical transformations like those from this activity are used.
Activity 1a:

Set 1:

\[ A - c d = c B \]
\[ A + f d = -f B \]

Set 2:

\[ \frac{1}{2} c d^2 = \frac{1}{2} c A^2 + c B^2 \]
\[ c d = c A + 2c B \]

Activity 1b:

\[ e^a e^{b+c} \]
\[ e^a e^{-(b+c)} \]
\[ e^{-a} e^{-(b+c)} \]
\[ \log(x) + 4 \log(y) \]
\[ \ln(x) - 4 \ln(x) \]
\[ \log \left( \frac{x}{4y} \right) + 4 \log(y) \]

Activity 1c:

\[ \log(B) = 4x \]
\[ \log(2B) = 6x \]
\[ \ln \left( \frac{1}{B} \right) = \frac{1}{3x} \]
\[ \ln \left( \frac{5}{B} \right) = \frac{1}{5x} \]
Activity 2a:

\[ y = Dx + F \]
\[ y = -Dx + F \]
\[ y = x^2 \]
\[ y = x^2 + b \]
\[ y = -x^2 + b \]
\[ y = -(x - c)^2 + b \]

Activity 2b:

\[ y = A \sin(x) \]
\[ y = A \sin(x) + b \]
\[ y = A \sin(x + c) + b \]
\[ y = A \cos(x) \]
\[ y = A \cos(x) + b \]
\[ y = A \cos(x + c) + b \]

Activity 2c:

\[ y = \frac{1}{x^2} \]
\[ y = I_0 e^x \]
\[ y = I_0 e^{-x} \]
Activity 3:

\[ h = g^2 + b \]
\[ h = \frac{1}{g^2} + bc^2 \]
\[ h = f g^3 + bc^2 \]
\[ \frac{1}{h} = \frac{1}{2} (f g)^3 + \frac{b}{c} \]
\[ f = 2 h (g^2 + b) \]

Determine for each equation:

- The label on the x-axis
- The label on the y-axis
- The slope
- The y-intercept