Functions

Overview

Description

Functions are one of the most important fundamental concepts in mathematics. In this activity, students will explore this concept from different perspectives.

Final Product: Students should understand what functions are and the different ways in which they can be represented.

Subject

Pre-Calculus

Task Level

Grade 11-12

Objectives

Students will:

- Review the definition of a function.

- Analyze function properties graphically and algebraically.

- Explore the equality of functions.

- Express given situations in terms of function.

Preparation

- Read the Instructor Task Information and the Student Notes.

- Review the definition and representations of functions.

Prior Knowledge

- Students should be able to carry out algebraic manipulations, understand the basic definition of a function, and understand how functions are represented graphically.

Key Concepts and Terms

- Domain

- Function

- Graph

- Inverse

- Range

Time Frame

This assignment will require approximately two to three class periods. Plan one class period for students to review the basic concept of functions and one or two class periods for the further exploration and presentation. This assignment can be modified to meet the needs of different classroom schedules and student ability levels.

Instructional Plan

Getting Started

Learning Objectives

Students will:

- Review the definition of a function.

Procedure

1. Ask students to define what a function is.

2. Discuss different answers, or ask students to seek definitions if they cannot remember.

3. Distribute the Student Notes.

Investigating

Learning Objectives

Students will:

- Understand the graphical representation of a function and be able to gather information from a graph.

- Gather information about a function given its algebraic representation.

- Extend their basic knowledge of functions to a greater conceptual understanding.

Procedure

1. Have students complete the Investigating portion of the activity in small groups. Students should write their own solutions.

Drawing Conclusions

Learning Objectives

Students will:

- Compare their answers to the activity.

Procedure

1. Have students present and justify their solutions.

2. When solutions appear different, have students determine whether their solutions are truly different or whether they are the same but appear different.

Scaffolding/Instructional Support

The goal of scaffolding is to provide support to encourage student success, independence, and self-management. Instructors can use these suggestions, in part or all together, to meet diverse student needs. The more skilled the student, however, the less scaffolding that he or she will need. Some examples of scaffolding that could apply to this assignment include:

- Have students define their own functions graphically, verbally, and algebraically.

- Have students express the functions they defined in other ways. For example, graph a function that they defined algebraically.

- Have students identify which of the functions they defined have inverses.

- Use questions such as these to prompt student thinking:

  o   Can you draw a picture of a function?

  o   What different ways can you represent a function?

  o   A function has inputs and outputs. For every input to a function, how many outputs are matched to the input?

  o   What are the set of values that are the input to a function called?

  o   What are the set of values that are the output of a function called?

  o   In a graphical representation of a function, on what axis are the values in the domain?

  o   In a graphical representation of a function, on what axis are the values in the range?

 o   Do two functions that are equal have the same domain and range?

Solutions

The information below is intended to help you assess students' final work products. It may not represent all possible strategies and ideas. The accompanying scoring guide provides specific examples of ways a student might demonstrate content understanding and mastery of cross-disciplinary skills.

Investigating

1. The domain is [0,5)   (5,9], and the range is [-2, 3]. The maximum and minimum values are 3 and -2, respectively. The function is increasing on the intervals [0,1], (2, 5), and [8,9]. It is decreasing on [1, 2] and (5, 8).

2. No: $f(3x) = 1 - (3x)^2 = 1 - 9x^2 \neq 3 - 3x^2 = 3f(x)$.

3. First, $f(11) = \sqrt[3]{11-3} = \sqrt[3]{8} = 2$.   Then, $g(11) = f(11) + 4 = 2 + 4 = 6$.

4. The domain is [0,5)   (5,∞).

5. The diagrams for *f* and *h* represent functions; only *h* has an inverse.

6. The inverse is $\dfrac{-3x-7}{5x-2}$.

7. They are equal in the first and fourth cases, and not in the others.

8. One possible pair of graphs:
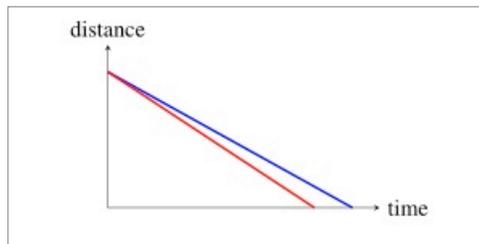
9. The graph is
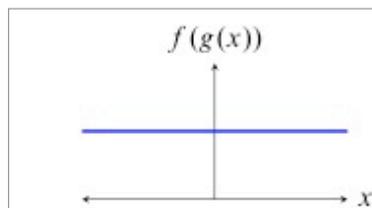
10. The area is $2x^2 + \dfrac{32}{x}$.

11. $f(n) = n(n-1)$

TCCRS Cross-Disciplinary Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Key Cognitive Skills			
A.1. Engage in scholarly inquiry and dialogue.	✓	✓	✓
A.2. Accept constructive criticism and revise personal views when valid evidence warrants.		✓	✓
B.1. Consider arguments and conclusions of self and others.	✓	✓	✓
B.2. Construct well-reasoned arguments to explain phenomena, validate conjectures, or support positions.	✓	✓	✓
B.4. Support or modify claims based on the results of an inquiry.	✓	✓	✓
D.1. Self-monitor learning needs and seek assistance when needed.	✓	✓	✓
D.3. Strive for accuracy and precision.	✓	✓	✓
D.4. Persevere to complete and master tasks.	✓	✓	✓
E.1. Work independently.	✓	✓	✓
E.2. Work collaboratively.		✓	✓
II. Foundational Skills			
B.1. Write clearly and coherently using standard writing conventions.			✓
C.8. Present final product.			✓

TCCRS Mathematics Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Numeric Reasoning			
B.1. Perform computations with real and complex numbers.	✓	✓	✓

II. Algebraic Reasoning			
B.1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).	✓	✓	✓
VII. Functions			
A.1. Recognize whether a relation is a function.		✓	
B.1. Understand and analyze features of a function.		✓	
C.2. Develop a function to model a situation.		✓	
VIII. Problem Solving and Reasoning			
A.1. Analyze given information.		✓	
C.2. Use a function to model a real world situation.		✓	

TEKS Standards Addressed

Functions - *Texas Essential Knowledge and Skills (TEKS): Math*
111.32.b.2. Foundations for functions. The student uses the properties and attributes of functions. The student is expected to: 111.32.b.2.B. identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete;
111.32.b.3. Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to: 111.32.b.3.A. use symbols to represent unknowns and variables.

111.32.b.5. Linear functions. The student understands that linear functions can be represented in different ways and translates among their various representations. The student is expected to:
        111.32.b.5.B. determine the domain and range for linear functions in given situations; and
        111.32.b.5.C. use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.

111.32.b.9. Quadratic and other nonlinear functions. The student understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions. The student is expected to: 111.32.b.7.A. determine the domain and range for quadratic functions in given situations;

| **Functions -** *Texas Essential Knowledge and Skills (TEKS): Math* |
|---|
| 111.33.b.1. Foundations for functions. The student uses properties and attributes of functions and applies functions to problem situations. The student is expected to:
      111.33.b.1.A. identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and |
| 111.33.b.4. Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to:
      111.33.b.4.C. describe and analyze the relationship between a function and its inverse. |
| 111.35.c.1. The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to:
      111.35.c.1.B. determine the domain and range of functions using graphs, tables, and symbols;
      111.36.c.1.C. recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function; and |
| 111.35.c.2. The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to:
      111.35.c.2.B. perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically; |

Functions

Introduction

Functions are one of the most important fundamental concepts in mathematics. In this activity, you will explore this concept from different perspectives.

Directions

Getting Started

1. This activity should be done without a calculator.

2. Review the definition of a function. Answer the following: What is a function? What is an inverse function? What does it mean for two functions to be equal?
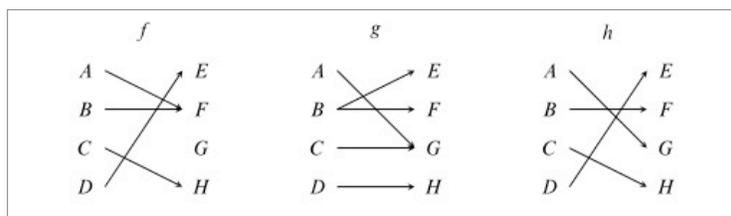
Investigating
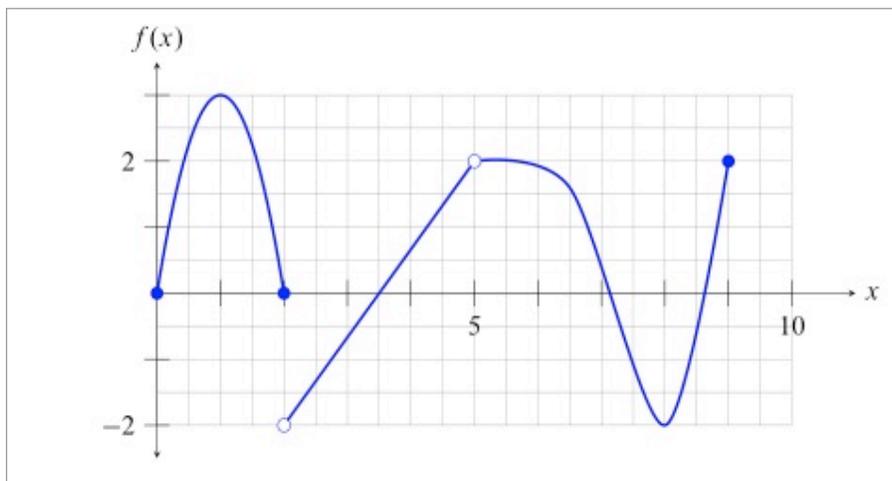
1. Consider the above graph of a function *f*.

    a. What is the domain?

    b. What is the range?

    c. What are the maximum and minimum values?

    d. On what interval(s) is *f* increasing? Decreasing?

2. Let $f(x) = 1 - x^2.$ Is $f(3x) = 3f(x)$?

3. Let $f(x) = \sqrt{x-3}$ and let $g(x) = f(x) + 4.$ Find $g(11).$

4. Find the domain of the function $f(x) = \dfrac{\sqrt{x}}{5-x}.$

5. Consider the following diagrams

   a. Which, if any of these, represent functions?

   b. Which, if any of these, represent functions that have inverses?

6. Find the inverse of the function $y = \dfrac{2x-7}{5x+3}$

7. Determine in each case whether f and g are equal to each other.

   a. $f(x) = x^2$ and $g(x) = 3 + x^2 - 3$

   b. $f(x) = \sqrt{x^2}$ and $g(x) = x$

   c. $f(x) = x$ and $g(x) = 1/(1/x)$

   d. $f(x) = 1/x$ and $g(x) = f^{-1}(x)$

8. Suppose that you are running a race, and that *f(t)* represents your distance from the finish line at time *t.*

   a. Sketch a possible graph of *f.*

   b. Suppose that your friend also ran the race, and that *g(t)* represents his distance from the finish line at time *t.* Sketch on the same axes a possible graph of *g* assuming that your friend won the race.

9. Consider the following graphs of function *f* and *g*.
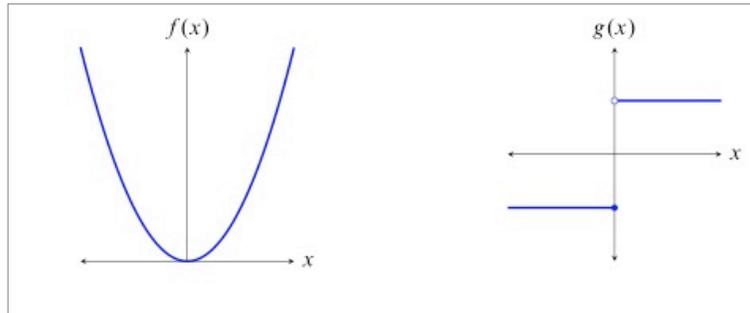
Sketch the graph of *f(g(x))*.

10. A closed rectangular box has a square base and a volume of 8 cubic meters. Express the outer surface area of the box as a function *S(x)* of the length *x* of a side of the base.

11. You attend a party, and you end up saying "hello" to everyone else exactly once. Assuming that everyone else did the same thing, define a function *f(n)* that represents the number of times "hello" is said at the party in terms of the number *n* of people there.

Drawing Conclusions

Compare your solutions to those of the other groups. Which are different? Are these differences real or apparent? In other words, which ones are actually different and which ones are the same solutions written in different forms?