Exponential and Logarithmic Functions

Overview

Description

Understanding exponential and logarithmic functions is essential before learning Calculus. In this activity, students will consider these functions graphically and algebraically, and will consider their connection to investment.

Final Product: Students should produce written answers. They should understand what exponential and logarithmic functions are and how they relate to each other, and the basics of exponential growth.

Subject

Pre-Calculus

Task Level

Grade 11-12

Objectives

Students will:

- Review the definition of an exponential and a logarithmic function.
- Analyze these functions' properties graphically and algebraically.
- Explore the equality of functions.
- Connect exponential functions to investment.

Preparation

- Read the Instructor Task Information and the Student Notes.
- Review the definition and representations of functions.

Prior Knowledge

Students should know what exponential and logarithmic functions are, be able to graph them, be able to carry out algebraic manipulations with exponential and logarithmic functions, and be able to recognize and solve problems deal with exponential growth.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Key Concepts and Terms

- Exponential
- Exponential growth
- Function
- Inverse
- Logarithms

Time Frame

Plan to take a week to complete this activity, depending on the amount of work assigned outside of class. Plan one class period for students to review exponential and logarithmic functions, two to three class periods to complete the Investigating portion of the activity, and one or two class periods to complete the Drawing Conclusions section.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Instructional Plan

Getting Started

Learning Objectives

Students will:

- Review the definition of a exponential and logarithmic functions
- Review the idea of inverse functions.
- Review the exponent and logarithm laws.

Procedure

- Ask students to define what an exponential function is.
- Ask students to define what an inverse function is.
- Ask students to define what a logarithmic function is.
- Discuss different answers, or ask students to seek definitions if they cannot remember.
- Distribute the Students Notes.

Investigating

Learning Objectives

Students will:

- Understand the graphical representation of translated exponential and logarithmic functions.
- Manipulate expressions using exponent and logarithm laws.
- Extend their basic knowledge of exponential and logarithmic functions to a greater conceptual understanding, including investment and exponential growth.

Procedure

1. Have students complete the Investigating portion of the activity in small groups. Students should write their own solutions.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Drawing Conclusions

Learning Objectives

Students will:

- Compare their answers to the activity.

Procedure

1. Have students present and justify their solutions.
2. When solutions appear different, have students determine whether their solutions are truly different or whether they are the same but appear different.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Scaffolding/Instructional Support

The goal of scaffolding is to provide support to encourage student success, independence, and self-management. Instructors can use these suggestions, in part or all together, to meet diverse student needs. The more skilled the student, however, the less scaffolding that he or she will need. Some examples of scaffolding that could apply to this assignment include:

- Ask students for examples of exponential and logarithmic behavior in the world.
- Review function domains as necessary.
- Discuss the origin of continuously compounded interest as a limit of periodically compounded interest.
- Have students use graphing software to explore the effects of changing the exponential and logarithmic parent functions.
- Have students create tables of values for the problems in the Investigating section of the activity, and ask them to graph, find an equation if one isn't given, or to verbally explain any patterns that they see in order to solve the problems.
- Have a group of students research and present a brief history of *e*.
- Have students describe the relationship between *e* and ln verbally and graphically.
- As students work through the Investigation and Drawing Conclusions sections of the assignment, observe the students by monitoring their progress. If students seem stuck, bring the class back together and discuss the concept the class is stuck on, or have a student or group present their solution and allow other students to ask questions.

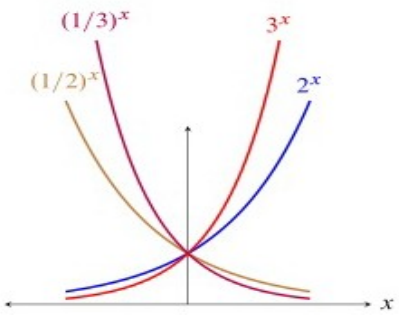
©2013 Texas Higher Education Coordinating Board. All rights reserved.

Solutions

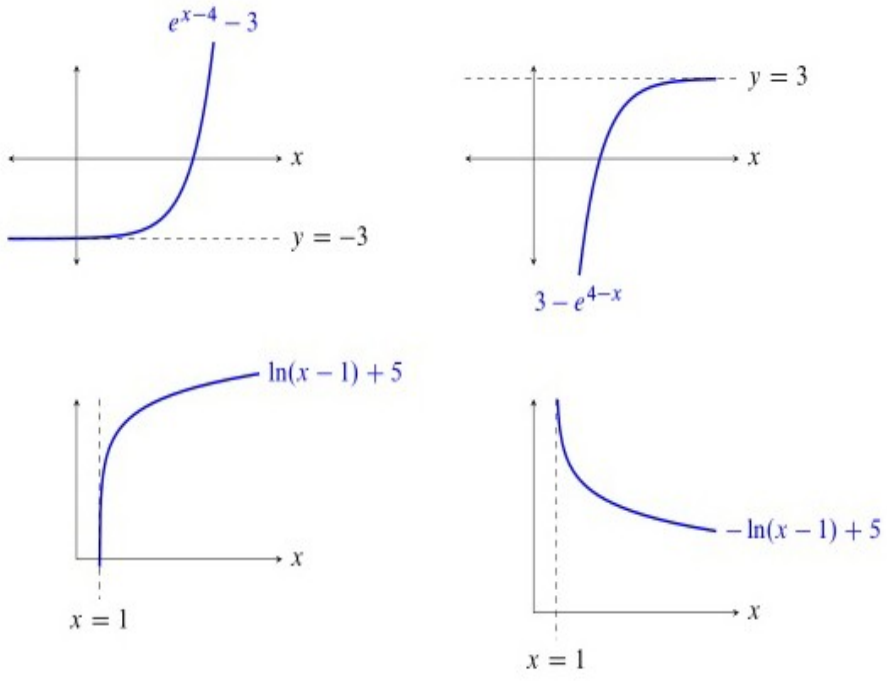
The information below is intended to help you assess students' final work products. It may not represent all possible strategies and ideas. The accompanying scoring guide provides specific examples of ways a student might demonstrate content understanding and mastery of cross-disciplinary skills.

Investigating

1. The graphs are:



2. The graphs are:



©2013 Texas Higher Education Coordinating Board. All rights reserved.

3. 1; 7.

4. Only (c).

5. No, their domains are different. (They are equal when both are defined.)

6. In all such cases, the amount is squared. This is not true with an initial Investment other than $1.

7. The penny option yields $21,474,863.47.

8. Its main strength is that the rate of growth increases as population increases. A weakness is that the population cannot increase exponentially forever, since it cannot increase without bound.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

TCCRS Cross-Disciplinary Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Key Cognitive Skills			
A.1. Engage in scholarly inquiry and dialogue.	✓	✓	✓
A.2. Accept constructive criticism and revise personal views when valid evidence warrants.		✓	✓
B.1. Consider arguments and conclusions of self and others.	✓	✓	✓
B.2. Construct well-reasoned arguments to explain phenomena, validate conjectures, or support positions.	✓	✓	✓
B.4. Support or modify claims based on the results of an inquiry.	✓	✓	✓
D.1. Self-monitor learning needs and seek assistance when needed.	✓	✓	✓
D.3. Strive for accuracy and precision.	✓	✓	✓
D.4. Persevere to complete and master tasks.	✓	✓	✓
E.1. Work independently.	✓	✓	
E.2. Work collaboratively.		✓	✓
II. Foundational Skills			
B.1. Write clearly and coherently using standard writing conventions.			✓
C.8. Present final product.			✓

TCCRS Mathematics Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Numeric Reasoning			
B. 1. Perform computations with real and complex numbers.		✓	
II. Algebraic Reasoning			

©2013 Texas Higher Education Coordinating Board. All rights reserved.

B. 1. Recognize and use algebraic (field) properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions (e.g., polynomials, radicals, rational expressions).	✓	✓	✓
VII. Functions			
B.1. Understand and analyze features of a function.		✓	
C.2. Develop a function to model a situation.		✓	
VIII. Problem Solving and Reasoning			
A.1. Analyze given information.		✓	
C.2. Use a function to model a real world situation.		✓	
IX. Communication and Representation			
A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.	✓	✓	✓
C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.			✓
X. Connections			
B.2. Understand and use appropriate mathematical models in the natural, physical, and social sciences.		✓	

TEKS Standards Addressed

Exponential and Logarithmic Functions - *Texas Essential Knowledge and Skills (TEKS): Math*
111.32.b.2. Foundations for functions. The student uses the properties and attributes of functions. The students is expected to: 111.32.b.2.B. identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.
111.33.b.2. Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to: 111.33.b.2.A. use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Exponential and Logarithmic Functions - *Texas Essential Knowledge and Skills (TEKS): Math*
111.33.b.4. Algebra and geometry. The student connects algebraic and geometric representations of functions. The students is expected to: 111.33.b.4.A. (A) identify and sketch graphs of parent functions, including linear (f (x) = x), quadratic (f (x) = x^2), exponential (f (x) = a^x), and logarithmic (f (x) = $\log_a x$) functions, absolute value of x (f (x) = lxl), square root of x (f (x) = sqrtx), and reciprocal of x (f (x) = 1/x);
111.33.b.11. Exponential and logarithmic functions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to: 111.33.b.11.A. develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses; 111.33.b.11.B. use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior; 111.33.b.11.C. determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities; 111.33.b.11.D. determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods; 111.33.b.11.E. determine solutions of exponential and logarithmic inequalities using graphs and tables; and 111.33.b.11.F. analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.
111.35.c.1. The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to: 111.35.c.1.A. describe parent functions symbolically and graphically, including f(x) = xn, f(x) = ln x, f(x) = loga x, f(x) = 1/x, f(x) = ex, f(x) = lxl, f(x) = ax, f(x) = sin x, f(x) – arcsin x, etc. 111.35.c.1.D recognize and use connections among significant values of a function (zeros, maximum values, minimum values, etc.), points on the graph of a function, and the symbolic representation of a function.
111.35.c.2. The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to: 111.35.c.2.A. apply basic transformations, including a * f(x), f(x) + d, f(x – c), f(b * x), and compositions with absolute value functions, including lf(x)l, and f(lxl), to the parent functions. 111.35.c.2.B. perform operations including composition on functions, find inverses, and describe these procedures and results verbally, numerically, symbolically, and graphically. 11.35.c.2.C. investigate identities graphically and verify them symbolically, including logarithmic properties, trigonometric identities, and exponential properties.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Exponential and Logarithmic Functions - *Texas Essential Knowledge and Skills (TEKS): Math*
111.35.c.3. The student uses functions and their properties, tools and technology, to model and solve meaningful problems. The student is expected to: 111.35.c.3.B. use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data.

©2013 Texas Higher Education Coordinating Board. All rights reserved.

Exponential and Logarithmic Functions

Introduction

Understanding exponential and logarithmic functions is essential before learning Calculus. In this activity, you will consider these functions graphically and algebraically and will consider their connection to investment.

Directions

This activity should be done without a calculator, except for problems 6 and 7.

Getting Started

Review the definition of exponential and logarithmic functions. Answer the following: What is an exponential function? What is a logarithmic function? How do these types of functions relate to each other graphically and algebraically?

Investigating

1. Sketch the graphs of the following functions on a single set of axes. Show correct intercepts with the axes, intersections of each graph with each other, and correct relative positions of each graph (e.g., a larger function should be above a smaller one).
   a) $f(x) = 2^x$
   b) $f(x) = 3^x$
   c) $f(x) = (1/2)^x$
   d) $f(x) = (1/3)^x$

2. Sketch the graphs of the following functions.
   a) $f(x) = e^{x-4} - 3$
   b) $f(x) = 3 - e^{4-x}$
   c) $f(x) = \ln(x - 1) + 5$
   d) $f(x) = -\ln|x - 1| + 5$

3. Simplify the following expressions.
   a) $\ln(e^e)/e$
   b) $\log_{31}(31^7)$

©2013 Texas Higher Education Coordinating Board. All rights reserved.

4. Which, if any, are true?

   a) $e^{\ln(-3)} = -3$

   b) $e^{\ln(-3)} = \ln(e^{-3})$

   c) $\ln(e^{-3}) = -3$

   d) $\ln(e^{-3})$ is undefined

5. Are $f(x) = e^{\ln(x)}$ and $g(x) = \ln(e^{x})$ equal?

6. Suppose that you have made a small investment of $1. The interest is compounded continuously at an annual rate of 2%, which means that the total amount after t years is given by the formula $\$1 \cdot e^{0.02t}$, or simply $e^{0.02t}$.

   a) How much does your investment grow between years 2 and 4? Between 4 and 8? In general, how much does the investment grow between any given time and when twice that amount of time has passed?

   b) Is this still true if you started with an investment of $10?

7. Suppose that a generous friend hires you to mow his lawn each day during July. He offers you two choices. You can either earn $1,000 per day, or you can start by earning a penny on the first day, and then each day thereafter you would earn twice what you earned on the previous day. Which offer would you choose?

8. Many natural phenomena are modeled using exponential functions. What are the strengths and weaknesses of exponential growth as a model for population growth?

Drawing Conclusions

Compare your solutions to those of the other groups. Which are different? Are these differences real or apparent? In other words, which ones are actually different and which ones are the same solutions written in different forms?

©2013 Texas Higher Education Coordinating Board. All rights reserved.