Checking the Numbers

Overview

Description

A Universal Product Code (UPC) is found on most items you buy. The code is given as a bar code for instantaneous machine reading. But if the machine reader fails, then the code may be entered by a human being as a string of 11 digits followed by one more digit, called the "check digit" number. These 11 digits along with the check digit are found just below the bar code itself. Since it is so easy to make a mistake when entering a string of numbers, the "check digit" number's purpose is to immediately reveal if an entering error has been made; the bar code can then be entered a second time for correction. A similar system is used for identifying books, called the International Standard Book Number (ISBN). This assignment gives students experience with these codes and teaches them how to recognize when entering errors occur.

Final Product: The student will answer in writing a series of questions demonstrating their understanding of bar codes, how to recognize when an error has been made in entering a bar code, and, in some cases, determining what the error was. They will also be asked to report briefly on outside research on the difference between the ISBN-13 number and the UPC code.

Course

Algebra I or II

Task Level

Grades 9-12

Objectives

Students will:

- Determine the check digit of a UPC identification number and an ISBN number.
- Analyze the effects of basic data entry errors.
- Analyze errors in recording UPC identification numbers and ISBN numbers detected by its check digit method.

Preparation

- Read the Instructor Task Information and the Student Notes.
- Prepare student copies of the Student Notes pages and the handout.

- Prepare a collection of items with UPC codes and books of various types for students to examine the UPC/ISBN identification number.
- Gather a handful of ten-key machines to allow students to practice entering numbers manually.

Prior Knowledge

Students should be able to combine algebraic expressions into altered forms and read meaning into the resulting expression. Calculations for the check-digit method can be performed easily on a calculator.

Key Concepts and Terms

- Check digit
- Identification number
- Single-digit error
- Transposition error

Time Frame

Plan at least one class period for investigating the detection of bar code errors. Then plan one additional class period for small groups to continue their investigation of UPC and ISBN error detection, and up to two homework periods to formally answer the questions asked and do the additional research required.

Instructional Plan

Getting Started

Learning Objectives

Students will:

- Understand the use of numbers and number checking in real life examples.
- Analyze basic data entry errors.

Procedure

1. Show students the ISBN number on their textbooks. Start the conversation about what the digits mean and why such a number is used on every book.
2. Have students practice entering the numbers on their own using the provided ten-key machines or computer keyboards.
3. Discuss with students their own errors in recording numbers. Ask what type of error(s) they make. Many may say entering the incorrect digit or transposing two digits—the two most common errors.

Investigating

Learning Objectives

- Students will analyze bar codes to identify single-digit and transposition errors.
- Students will apply mathematical strategies for estimation to identify errors.

Procedure

1. After verifying their ability to calculate a check digit for a UPC number and an ISBN number, students can work in small groups to discover the errors that are and are not detected by these methods. Students can share their small group work with the whole class during discussion to solidify new learning.
2. You may wish to show students a method to find a UPC check digit and ISBN check digit using their calculators. The method for the UPC numbers is shown in the *Supplementary Materials*; the method for ISBN numbers is similar.

Drawing Conclusions

Learning Objectives

Students will:

- Students will apply mathematical concepts to real life issues.

- Students will use data and mathematical reasoning to explain their answers.

Procedure

1. Help students understand that identifying the position in which a single-digit error is made is not possible. Only ten check digits are possible and there are millions of correct bar codes.
2. In general, students will probably need some demonstrated instruction to validate the conjecture that the UPC method detects all single-digit errors.
3. Walk students through the questions they are being asked to answer in writing, including the research question.

Scaffolding/Instructional Support

The goal of scaffolding is to provide support to encourage student success, independence, and self-management. Instructors can use these suggestions, in part or all together, to meet diverse student needs. The more skilled the student, however, the less scaffolding that he or she will need.

- If students are struggling to understand UPC codes and check digits, do a couple example problems for the class and then have the students solve a problem individually and ask a student to share how he or she solved the problem. Choose from the problems below.
- It may be helpful to work through this assignment by switching between full class discussion and small groups and breaking the assignment up in to smaller chunks by giving students one, or a couple of questions to work on at a time. This will help the students move through the assignment at roughly the same pace.

The following are some scaffolding problems for the *Checking the Numbers* assignment:

- Calculate the check digit D for the bar code: 12345 67890 D.
- Calculate the check digit value D for each of the bar codes: 10101 010101 D and 01010 101010 D.
- Calculate the check digit D for the bar code: 13131 313131 D. Calculate D for the code 31313 131313 D.
- Give three specific, but different, bar codes that have three different check digit values,
- How many possible check digit values are there in the universe? How many bar codes are possible in the universe?
- Give three specific, but different bar codes that have the same check digit value of D = 7; that have the check digit value of D = 0.

Solutions

The solution provided in this section is intended to clarify the problem for teachers. This solution may not represent all possible strategies for approaching the problem or all possible solutions. It should be used for reference only.

The following solutions belong to the questions in the Student Notes.

Checking the Numbers

1. a) *$8000*

   b) *$1*

2. 1000a + 100b + 10c + d – (1000a + 100b + 10d + c) = 9(c – d)

   1000a + 100b + 10c + d – (1000a + 100c + 10b + a) = 90(b – c)

   1000a + 100b + 10c + d – (1000b + 100a + 10c + d) = 900(a – b)

   All three differences are divisible by 9.

Universal Product Code [UPC] Numbers

1. D = 1 because

   3(3+ 1+ 7 + 0 + 2 + 3) + (8 + 3 + 0 + 9 + 1+ D) =

   3(16) + 21 + D =

   69 + D is 70 when D=1.

   a) S + D = 3(0 + 2 + 0 + 4 + 8 + 6) + (5 + 2 + 0 + 2 + 8 + 5) = 4(20) + 2 which is not a multiple of 10.

   b) No. Any single digit can be increased or decreased by an amount that would cause the sum of the resulting twelve digits to end in a multiple of 10.

   c) Any of the first 11 digits could be changed; for example, 4 could be changed to 6.

5. a) *3(0 + 8 + 0 + 4 + 8 + 6) + (5 + 2 + 0 + 2 + 8 + 5) = 3(26) + 22 = 100, which ends in 0.*

   b) The difference between the two code numbers is 3(7) – 3(6) = 3, so the sum of the weighted sum of the digits in the invalid number could not end in 0.

   c) Yes: To show that all single digit errors are detected, suppose the digit b is recorded instead of the correct digit a in an even-numbered position in an UPC code number. By the UPC method, we have

   $3a_1 + a_2 + 3a_3 + a\ldots + d$ is a multiple of 10

   $3a_1 + a_2 + 3a_3 + b\ldots + d$ is a multiple of 10

   Suppose no error was detected. Then the difference between these two numbers, a–b, is also a multiple of 10 – which is not possible since a and b are different digits. Thus UPC method will detect the error.

Now suppose the digit b is recorded instead of the correct digit a in an odd-numbered position in an UPC code number. By the UPC method, we have

$3a_1 + a_2 + 3a_3 + 3a\ldots + d$ is a multiple of 10

$3a_1 + a_2 + 3a_3 + 3b\ldots + d$ is a multiple of 10

Suppose no error was detected. Then the difference between these two numbers, $3a - 3b = 3(a - b)$, is also a multiple of 10 which means that $a - b$ is a multiple of 10. Again this is not possible since a and b are different digits, so the UPC method will detect the error.

6. All single-digit errors will be detected. This argument in #6b works for the weights

   {9, 1, 9, 1, … , 9,1}

7. If the positive difference between the incorrect digit and the correct digit is 5, the error will not be detected. For an odd-numbered position, for example, modifying the argument in #6b, if $2a - 2b$ is a multiple of 10, then a and b could differ by 5 and such a single-digit error would not be detected.

8. If $|a - b| = 5$, the error will not be detected because $3(a-b)-(b-a)=4(b-a)$.

International Standard Book Numbers (ISBNs)

9. D = 7

10. The digits 3 and 9 are transposed. The correct number is 0-669-09325-4

11. The check digit is the same.

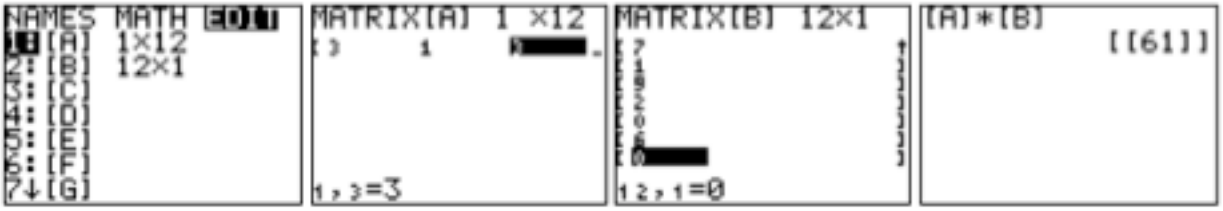
12. a) If there is an error in the k^{th} place, say b for a, then $kb - ka = k(b - a)$ is divisible by 11 which forces $a = b$ since $1 < k < 10$

    b) Yes

Supplementary Materials

1. After verifying their ability to calculate a check digit for a UPC number and an ISBN number, students can work in small groups to discover the errors that are and are not detected by these methods. Students can share their small group work with the whole class during discussion that solidify new learning.
2. You may wish to show students a method to find a UPC check digit and ISBN check digit using their calculators. The method for the UPC numbers is shown – the method for ISBN numbers is similar. Show students that they can use matrix methods to calculate check digits. To determine the check digit using the UPC method:
    1. Enter the weights {3, 1, 3, 1, ….., 1} in a 1x12 matrix, A.
    2. Enter the first 11 digits of the UPC number and "0" as the 12th digit in a 12x1 matrix, B.
    3. Compute [A] * [B]. If the result ends in a "0" then 0 is the check digit. If the result ends in U, then (10 – U) is the check digit.

       For example, the UPC for the TI-84 Plus Silver Edition Graphing Calculator is 03331719206D. The first 6 digits, 033317, is the Texas Instruments! company prefix, 9206 identifies the specific calculator, and D is the check digit.
    4. The matrix product is 61 which ends in 1, so the check digit is 10 – 1 = 9.



Help students understand that identifying the position in which a single digit error is made is not possible – any digit can be changed to produce a valid UPC number.

To validate the conjecture that the UPC method detects all single-digit errors, in general, students will probably need some direct instruction. Here is a possible explanation to present:

To show that all single digit errors are detected, suppose the digit b is recorded instead of one of the correct digit a in an **even-numbered position** in an UPC code number. By the UPC method, we have:

$3a_1 + a_2 + 3a_3 + \mathbf{a}\ldots + d$ is a multiple of 10

$3a_1 + a_2 + 3a_3 + \mathbf{b}\ldots + d$ is a multiple of 10

Suppose no error was detected. Then the difference between these two numbers, **a** – **b**, is also a multiple of 10 – which is not possible since a and b are different digits. Thus the UPC method will detect the error.

Now suppose the digit b is recorded instead of the correct digit a in an **odd-numbered position** in an UPC code number. By the UPC method, we have

$3a_1 + a_2 + 3a_3 + \mathbf{3a}\ldots + d$ is a multiple of 10

$3a_1 + a_2 + 3a_3 + \mathbf{3b}\ldots + d$ is a multiple of 10

Suppose no error was detected. Then the difference between these two numbers, $3a - 3b = 3(a - b)$, is also a multiple of 10 which means that $a - b$ is a multiple of 10. Again this is not possible since a and b are different digits, so the UPC method will detect the error.

This type argument works for the weights {9, 1, 9, 1, … , 1} in #7 and for ISBN numbers in #12 and #13

TCCRS Cross-Disciplinary Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Key Cognitive Skills			
A.1. Engage in scholarly inquiry and dialogue.	✓		✓
B.2. Construct well-reasoned arguments to explain phenomena, validate conjectures, or support positions.		✓	✓
B.3. Gather evidence to support arguments, findings, or lines of reasoning.		✓	✓
C.2. Develop and apply multiple strategies to solve problems.	✓		✓
D.1. Self-monitor learning needs and seek assistance when needed.	✓	✓	✓
D.2. Use study habits necessary to manage academic pursuits and requirements.	✓	✓	✓
D.3. Strive for accuracy and precision.	✓	✓	✓
D.4. Persevere to complete and master tasks.	✓	✓	✓
E.1. Work independently.	✓	✓	✓
II. Foundational Skills			
B.1. Write clearly and coherently using standard writing conventions.	✓	✓	✓
D.1. Identify patterns or departures from patterns among data.	✓	✓	✓
D.3. Present analyzed data and communicate finding in a variety of formats.	✓		

TCCRS Mathematics Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Numeric Reasoning			
A.1. Compare real numbers.	✓	✓	
C.1. Use estimation to check for errors and reasonableness of solutions.		✓	

II. Algebraic Reasoning			
B.1. Recognize and use algebraic properties, concepts, procedures, and algorithms to combine, transform, and evaluate expressions.			✓
VIII. Problem Solving and Reasoning			
A.1. Analyze given information.		✓	✓
A.2. Formulate a plan or strategy.			✓
A.3. Determine a solution.			✓
A.4. Justify the solution.			✓
IX. Communication and Representation			
A.1. Use mathematical symbols, terminology, and notation to represent given and unknown information in a problem.	✓	✓	✓
A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.			✓
A.3. Use mathematics as a language for reasoning, problem solving, making connections, and generalizing.			✓
X. Connections			
A.2. Connect mathematics to the study of other disciplines.	✓	✓	✓

TEKS Standards Addressed

Checking the Numbers - *Texas Essential Knowledge and Skills (TEKS): Math*
111.32.b.3. Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. The student is expected to: 111.32.b.3.A. use symbols to represent unknowns and variables. 111.32.b.3.B. look for patterns and represent generalizations algebraically.
111.32.b.4. Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to: 111.32.b.4.A. find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations. 111.32.b.4.B. use the commutative, associative, and distributive properties to simplify algebraic expressions.

Checking the Numbers - *Texas Essential Knowledge and Skills (TEKS): Math*
111.36.c.1. The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to: 111.36.c.1.A. compare and analyze various methods for solving a real-life problem 111.36.c.1.B. use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines. 111.36.c.1.C. select a method to solve a problem, defend the method, and justify the reasonableness of the results.

Checking the Numbers

Introduction

In this assignment you will work with numbers that are very familiar to you: the numbers you have seen on UPC barcodes or ISBN book numbers. Retailers use these numbers daily, but how do they ensure that these numbers are entered correctly when an item is purchased? You will use a systematic method and mathematical principles to easily identify common errors in these frequently used numbers. You will provide written as well as mathematical answers to all questions included in the three stages of this assignment, including the extension, which will require some out-of-class research.

The Problem

Anytime you enter data in a checkbook, a document, a spreadsheet, etc., you may make an error. The most common error, by far, is a single-digit error, for example entering an 8 instead of a 3 somewhere in the number. Another common error is a transposition error where you reverse the order of two adjacent digits, writing “83” instead of “38.” Is there a way to use mathematical principles to easily identify when errors occur?

Directions

Getting Started

1. Review the *Checking the Numbers* handout.
2. Suppose you are balancing your checkbook. For simplicity, suppose all amounts in this checkbook are in dollars (no cents) from $1 to $9999.
   a. If you make one single-digit error, what is the greatest possible difference between your total of the numbers you have entered and the bank’s total?
   b. What is the least possible difference?
3. A common rule for spotting a transposition error is: “if the difference between your total and the bank’s total is divisible by 9, look for a transposition error.” Using algebra, explain why this rule makes sense.

   Hint: First consider the case where the transposition error is in the last two digits. If the correct amount of a check is 1000a + 100b + 10c + d, then the incorrect amount entered will be 1000a + 100b + 10d + c. What do you notice about the result of subtacting these two numbers? Then analyze the other two cases.

Investigating

Begin by first reading and understanding the definition of the “check digit” number used in a UPC barcode, found at the end of these notes.

1. Determine the check digit, D, for the UPC number 38137009213D.
2. Show that 0-52200-48826-5 is an invalid UPC Code number
3. If someone made a single-digit error in entering this invalid number, can you tell which digit is incorrect? Why or why not?
4. Change one digit (not the check digit) in this invalid number so the resulting number is valid.
5. Show that 0-58200-48826-5 is a valid UPC Code number. Suppose you entered an "7” instead of the "6” when recording this UPC number. Explain why the UPC method will detect this error.
6. Do you think the UPC method will detect all such single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.
7. Suppose you used the weights {9,1,9,1,9,1, 9,1} instead of {3,1,3,1,3,1, 3, 1} in the UPC method; that is, replace multiples of 3 with instead multiples of 9 to define the “check digit”. Do you think this method will detect single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.
8. Suppose you used the weights {2,1,2,1,2,1, 2, 1} instead of {3,1,3,1,3,1, 3, 1} in the UPC method. Do you think this method will detect single-digit errors? Either give several examples of numbers with an error that will not be detected or explain why you think all such single-digit errors will be detected.
9. In entering UPC code numbers, will the error of transposing 3 and 8 be detected? [i.e.……83… is recorded as ….38….] How about 3 and 7? Make a conjecture about the values of a and b for which the error of transposing ab and ba is detected by the UPC method. Explain your reasoning in making your conjecture.

Drawing Conclusions

1. Find the check digit D for the ISBN number 0-394-84494-D.
2. The incorrect ISBN number 0-669-03925-4 is the result of transposing two adjacent digits not involving the first or last digit. Find the correct ISBN number. Explain your reasoning.
3. Suppose, instead, that the check digit a10 is chosen so that the sum a1 + 2a2 + ... + 10a10 is evenly divisible by 11. How does the “check digit” determined

using this calculation compare to the one found using the given method? Explain your answer.

4. Explain why the ISBN method detects all single digit errors.
5. Does it also detect all transposition of adjacent digits error? Explain your reasoning in each case.
6. Extension: Do some research on the ISBN-13 number. In what ways is it similar to a UPC code number? In what ways is it different?

Checking the Numbers

Anytime you enter data in a checkbook, a document, a spreadsheet, etc., you might make an error. The most common error, by far, is a single-digit error; entering an 8 instead of a 3 somewhere in the number, for example. Another common error is a transposition error where you reverse the order of two adjacent digits; writing "83" instead of "38."

One method of identifying data entry errors is through a check digit – a digit appended to the end of the identification number for the purpose of detecting single-digit, transposition, and other types of errors. A check digit is standard for identification numbers such as airplane ticket numbers, ISBNs, UPC barcodes, credit card numbers, checking account numbers, passport numbers, VINs (vehicle identification numbers), POSTNet codes (bar codes from mailing addresses), etc. Older identification numbers such as social security numbers do not have a check digit.

Universal Product Code [UPC] Numbers

UPC code numbers are used to identify retail products. They are typically barcodes.

Source: www.systemid.com/learning_center/upc_code.jpg

In a 12-digit UPC bar code there are three main parts:

- Company prefix
- Item reference number
- Check digit

The company prefix is the first 6 digits of the UPC barcode. For the Coca-Cola Company, for example, the company prefix is 049000. This number will be the first 6 digits of the barcode for all Coca-Cola products. Company prefixes are assigned and licensed to companies by the Uniform Code Council (UCC). The company is a member of the UCC and pays an annual fee to the UCC (currently based on its gross sales) to license this unique number.

The item reference number follows. It has 5 digits. It identifies the specific product and its size, color, flavor, and so on.

The check digit is the 12th number. In the example above, the check digit is 7.

The Check Digit

When the bar code is entered by the bar reader, or by a human being, the machine instantly makes the following calculation:

Add the first, third, fifth, seventh, ninth and eleventh numbers.

Multiply this result by 3.

Add to this result the sum of the second, fourth, sixth, eight and tenth numbers.

Call this sum *S*.

For the UPC code digit $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}D$, the number *S* is:

$$S = 3a1 + a2 + 3a3 + a4 + 3a5 + a6 + 3a7 + a8 + 3a9 + a10 + 3a11$$

Note: *S* is an example of a *weighted sum* with weights (3,1,3,1,3,1,3,1,3,1,3).

For the example shown above (81123400000), verify that the sum *S* = 43.

What then is "*D*", the check digit number for this code? Informally, *D* is the number that you must add to *S* to reach the next highest multiple of 10 above *S*. For the bar code above, since S = 43, *D* must be 7, since 43 + 7 = 50, the next multiple of 10.

The formal definition of *D* is this: If *r* is the remainder when *S* is divided by 10, then $D = 10 - r$ unless $r = 0$, in which case $D = 0$. Thus, since 43 = 10(4) + 3, the remainder is 3, and the check digit *D* is (again) 10 - 3 = 7.

An equivalent alternative definition for the check digit *D* for a code with sum *S* is that number *D* such that *S* + *D* is a multiple of 10.

Suppose that a person entered, instead, 811324000007. The number *S* for this code is *S* = 41, which *should have* a check digit of 9. So the operator or the computer immediately knows that an error has been made because the code gives the check digit to be 7.

International Standard Book Numbers (ISBNs)

Another barcode system is the International Standard Book Number, or ISBN. This system was developed in the late 1960's and early 1970's so that every book could have a special 10-digit identification number.

This ten-digit number is composed of blocks of numbers identifying the language or country, the publisher, and the item number or edition for that publisher. The tenth digit is the check digit.

Beginning January 1, 2007, the ISBN system was replaced by the ISBN-13 system. It is a 13-digit number beginning with 978, followed by the current nine digits of the ISBN, and it will have a new check digit. When all old ISBNs have been used, the next series will begin with 979.

The figure shows an example of a 10-digit number and a 13-digit number. The check digit is determined using a method similar to the one used to determine the check digit for a UPC number.

For the older 10-digit numbers, the check digit is calculated by multiplying the first digit by 10, the second by 9, the third by 8, and continuing until the ninth digit is multiplied by 2. The sum of those products and the check digit must be a multiple of 11. That is, the check digit is the number that is added to the sum of products to reach the next highest multiple of 11.

One problem that arises in this process is that the check digit might need to be a 10. Because we only have digits 0-9, an X is written in the check-digit place in this case. (X is the Roman numeral for 10.)