# Changing Effects

## Overview

### Description

Students work in small groups to create a scenario in which the standard (or metric) units must be converted to a newly invented system of measurement that students create. Students then collect data from an online source (for example, http://www.weatherchannel.com) that applies to the scenario, and convert the data to their new system of measurement. Students conduct a statistical analysis on the converted data, and then conduct the same statistical analysis on the original data set. Finally, students Interpret and compare statistical results based on data using different units of measure, and consider the implications for science conducted globally.

**Final Product:** An individual report to a general audience in which students convey a) the scenario necessary for inventing the imaginary "new" system and how it impacted the units of measurement chosen, b) the meaning behind the statistical analyses conducted, c) the conclusions drawn about the comparative use of one system to another, and d) the significance of these conclusions in the larger scope of scientific inquiry.

### Subject

Algebra II, Mathematical Models with Applications

### Task Level

Grades 10–12

### Objectives

Students will:

- Create a hypothetical scenario requiring the use of data, statistical analysis, and conversion of units to a new system previously unused in the global arena.
- Interpret and compare statistical results based on data using different units of measurement.
- Conclude that some statistical measurements (z-scores, correlation) are not affected when measurement units are converted.
- Make a statement regarding the significance of this conclusion within the scope of global scientific inquiry.

## Preparation

- For students struggling to find appropriate data sets online, be prepared with some suggestions for accessible websites.
- Arrange access to graphing calculators and computers with *Fathom*® or Microsoft Excel software (or another statistics program).
- Prepare a copy of the Student Notes for each student.

## Prior Knowledge

Students will need experience with the following:

- *Fathom*®, Excel, or a similar program that will allow them to manage and analyze data, and present it in graphic form.
- The difference between the metric and standard systems of measurement, the origins of each and where they are used, and conversion between and within measurements systems
- Computing the mean, median, lower and upper quartiles, range, standard deviation, interquartile range (IQR), and z-scores for a given data set.
- Creating a scatterplot for paired data and determining the least-squares regression line and correlation coefficient.
- Selecting the appropriate measure of central tendency or range to describe a set of data and justify the choice for a situation.
- Drawing conclusions and making predictions by analyzing trends in scatterplots.
- Selecting and using an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology.
- Evaluating methods of sampling to determine validity of an inference made from a set of data.
- Recognizing misuses of graphical or numerical information, and evaluating predictions and conclusions based on data analysis.

## Key Concepts and Terms

- Boxplot, dotplot, histogram, scatterplot
- Confounding variables (or lurking variables)
- Correlation
- Experimental studies, observational studies
- Interquartile range
- Least-Squares regression equation, correlation coefficient

- Location statistics
- Mean
- Metric units, standard units, unit conversions
- Min/max, lower and upper quartiles, median
- Slope
- Standard deviation
- Statistical conclusion
- Univariate and bivariate quantitative data
- Variation statistics
- *Z*-score

## Time Frame

Plan one class period of 30-45 minutes for students to research appropriate data sets. Plan one week for students to complete the small group work and independently write up their conclusions based on their investigations.

# Instructional Plan

## Getting Started

### *Learning Objectives*

Students will:

- Theorize how yet a third system of measurement could develop and create a hypothetical “new” unit of conversion.
- Locate an appropriate data set online and develop a scenario that would require the conversion of the data to this new system.

### *Procedure*

1. Facilitate a class discussion to elicit this information:
   - The United States typically uses English units (i.e., pound, foot) when reporting measurements.
   - Other countries use metric units of measurement (centimeters, kilograms).
   - In our global economy, communication combined with speed and accuracy of sharing information is vital.
   - Countries use a myriad of different monetary units (and languages) while, globally, we use only two systems of measurement for height, weight, volume, etc.
2. Distribute the Student Notes and go over the Introduction and step 2 of Getting Started. Explain that after students work in small groups on the investigation, students will work individually with their group’s findings to draw conclusions and communicate them in a written report. Stress that it will be important to record the work in the investigation carefully and in a way that each member of the group will be able to access for their individual work.
3. Put students in groups of 2 to 3 for their work contemplating:
   - An imaginary scenario involving a new system of measurement.
   - Data that would be relevant to the scenario and comparable to the standard or metric system.

## Investigating

### *Learning Objectives*

Students will:

- Utilize online resources to locate a data set appropriate to an imagined scenario.

- Convert the data set to the units of an invented measurement system.
- Conduct statistical analyses on the converted data and the original data.
- Convert their data set to their invented system of measurement.
- Interpret and compare statistical results based on data using different units of measure.

### *Procedure*

1. Before small groups start to work, remind them that they need to record their work in a way that all members of the group can access when they go to write individual reports.
2. As small groups work, give students plenty of time to grasp the problem and think creatively about a scenario.
3. Deciding what methods and tools to use to convert data sets (or a sample of a data set) to the invented systems is an important part of this task. Let students make these decisions themselves in their groups.

## Drawing Conclusions

### *Learning Objectives*

Students will:

- Conclude that certain statistical measurements (*z*-scores, correlation) are not affected when measurement units are converted.

### *Procedure*

1. Remind students that they will work independently on this last part of the activity. They will consider the questions posed and then write up a report of their own thinking.

# *Scaffolding/Instructional Support*

The goal of scaffolding is to provide support to encourage student success, independence, and self-management. Instructors can use these suggestions, in part or all together, to meet diverse student needs. The more skilled the student, however, the less scaffolding that he or she will need. Some examples of scaffolding that could apply to this assignment include:

- Asking student to practice converting metric units to standard or customary units by hand, then using calculators or Excel.
- Having students spend some time working with a system that naturally uses a non-standard system of units: in finance one uses "basis points"; in the life sciences one sometimes uses a "generation" as the basic unit rather than years; in geometry one uses "radian measure" rather than degrees for circular measure, and then it turns out that 2pi-radians is the natural unit measure, rather than one radian; or logarithmic scaling that linearizes exponential data by treating doubling-time or half-life as the basic unit; then there is the non-linear AM radio dial, compared to the linear FM dial and this is surely related to the "A" and "F."
- Checking a student's assignment more frequently during its progress.
- Providing some students with additional review for specific statistical knowledge and skills (i.e., constructing a histogram to interpreting the graphical display, the differences between mean and median and when one central tendency measurement should be used over the other, etc.).
- Providing students with click sheets when using software such as Excel or Fathom.
- Providing suggestions of websites for students who struggle to find appropriate data.

# *Solutions*

The solution provided in this section is intended to clarify the problem for teachers. This solution may not represent all possible strategies for approaching the problem or all possible solutions. It should be used for reference only.

Global warming is a serious threat to our environment and future. Using a data set from *Fathom*® software, our data represents 45 years of measurements collected since 1950. Notice that the data for total carbon has been measured in metric tons per year. I will convert the total carbon dioxide emission measurements from metric to my units, "carbers" and compare how the statistical results vary. The conversion factor is 1 carber to 125 million metric tons.

GLOBAL CARBON (45 cases, 8 attributes)

The data below report carbon dioxide emissions (the amount we put into the atmosphere as opposed to how much we find there) from various sources around the world, each year from 1950 to 1994.

The file GlobalCarbon lumps the whole world together; the other–RegionalCarbon–separates the world into different regions, so you can see how the trends are different in different parts of the globe.

Each file reports the total emissions in metric tons (1 metric ton = 1000 kilograms) of carbon dioxide and breaks the total down into five sources: solid (e.g., coal), liquid (e.g., petroleum), gas (e.g., natural gas), cement (I didn't know cement production made carbon dioxide, did you?), and gas flaring (burning of gas at refineries and drilling sites to relieve excess pressure). The file also lists per-capita carbon dioxide production in metric tons per person per year.

- year: the year
- total: total carbon in MTons per year
- gas: from gas (e.g., natural gas)
- liquid: from liquid (petrol)
- solid: from solid (coal, wood)
- cement production
- gas flaring
- percap: per capita in Met Tons per year

SOURCE: Carbon Dioxide Information and Analysis Center
http://cdiac.esd.ornl.gov/ndps/ndp030.html

Here is a sample of the data set from *Fathom®* that I analyzed.

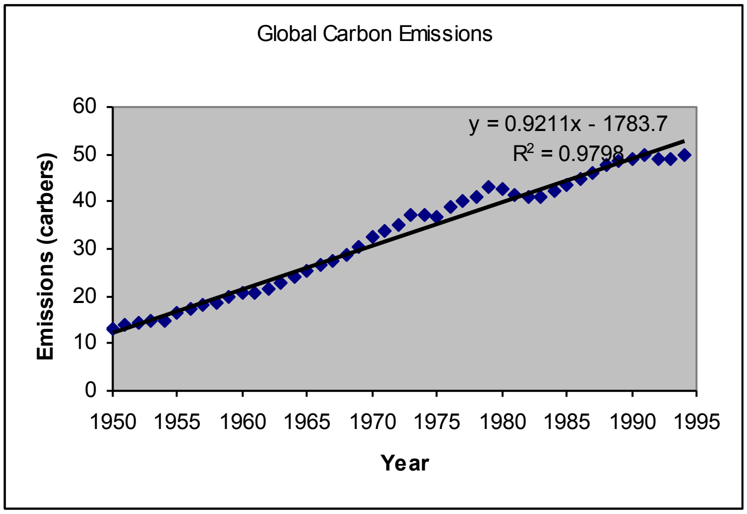
globalCarbon

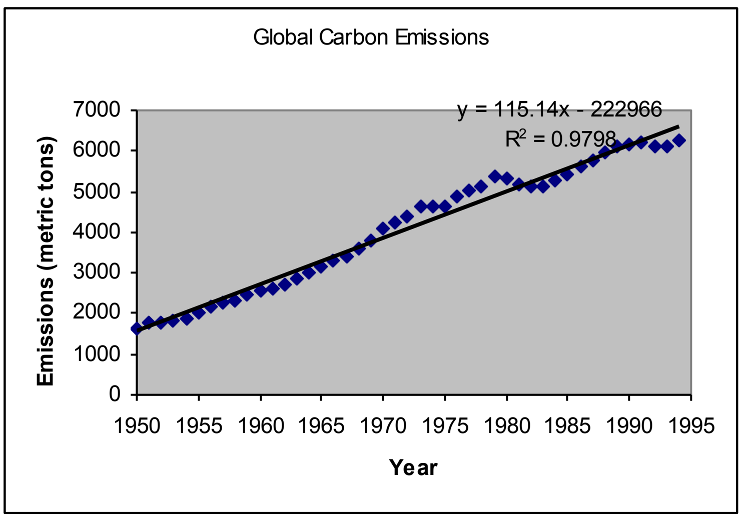
| | year | total | gas | liquid | solid | cement | flaring | percap | totalcustomary |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1950 | 1638 | 97 | 423 | 1077 | 18 | 23 | 0.65 | 1805.59 |
| 2 | 1951 | 1775 | 115 | 479 | 1137 | 20 | 24 | 0.69 | 1956.6 |
| 3 | 1952 | 1803 | 124 | 504 | 1127 | 22 | 26 | 0.69 | 1987.47 |
| 4 | 1953 | 1848 | 131 | 533 | 1132 | 24 | 27 | 0.7 | 2037.07 |
| 5 | 1954 | 1871 | 138 | 557 | 1123 | 27 | 27 | 0.69 | 2062.42 |
| 6 | 1955 | 2050 | 150 | 625 | 1215 | 30 | 31 | 0.74 | 2259.74 |
| 7 | 1956 | 2185 | 161 | 679 | 1281 | 32 | 32 | 0.78 | 2408.55 |
| 8 | 1957 | 2278 | 178 | 714 | 1317 | 34 | 35 | 0.8 | 2511.07 |
| 9 | 1958 | 2338 | 192 | 732 | 1344 | 36 | 35 | 0.8 | 2577.2 |
| 10 | 1959 | 2471 | 214 | 790 | 1390 | 40 | 36 | 0.83 | 2723.81 |
| 11 | 1960 | 2586 | 235 | 850 | 1419 | 43 | 39 | 0.86 | 2850.58 |

I converted the emissions data into carbers; the results are in the table below.

| Year | Metric tons | Carbers |
|---|---|---|
| 1950 | 1630 | 13.04 |
| 1951 | 1768 | 14.144 |
| 1952 | 1796 | 14.368 |
| 1953 | 1841 | 14.728 |
| 1954 | 1865 | 14.92 |
| 1955 | 2043 | 16.344 |
| 1956 | 2178 | 17.424 |
| 1957 | 2270 | 18.16 |
| 1958 | 2330 | 18.64 |
| 1959 | 2462 | 19.696 |
| 1960 | 2577 | 20.616 |
| Mean | 4707 | 37.7 |
| Standard Deviation | 1821 | 14.6 |

I used time (in years) as my independent variable and total carbon (metric tons per year and carbers per year) as my dependent variable to calculate the mean, standard deviation, LSR equation, coefficient of determination, and graphical displays using Microsoft Excel. In the table above, the mean and standard deviation are for all years from 1950 to 2006 rather than just the few years shown.




Comparing the statistical results:

I noticed that the LSR equations for carbers versus metric are different. However, if I convert the slope and y-intercept for the regression lines with the same factor of 125 I used for unit conversion, then the LSR equations do become the same. I also noticed that I can convert the mean and standard deviation in the same way. Furthermore, the correlation coefficients are the same, so both data sets have the same linear association between time and total carbon dioxide regardless of the units of measure.

Although the data look very different in raw form, the relationships within the original data and the residuals are clearly the same. This leads me to believe that although the original data and the converted data are different, they are, in fact, showing the same information. This would be true for any type of conversion, even to a new system that has not been invented yet or to an existing system developed by a culture that is not currently visible in the global arena.

This is significant because it allows scientists in different parts of the world to compare their findings as long as the conversions are done accurately.

## TCCRS Cross-Disciplinary Standards Addressed

| Performance Expectation | Getting Started | Investigating | Drawing Conclusions |
|---|---|---|---|
| *I. Key Cognitive Skills* | | | |
| A.1. Engage in scholarly inquiry and dialogue. | ✓ | ✓ | ✓ |
| B.2. Construct well-reasoned arguments to explain phenomena, validate conjectures, or support positions. | ✓ | ✓ | ✓ |
| C.3. Collect evidence and data systematically and directly relate to solving a problem. | ✓ | ✓ | ✓ |
| D.1. Self-monitor learning needs and seek assistance when needed. | ✓ | ✓ | ✓ |
| D.3. Strive for accuracy and precision. | ✓ | ✓ | ✓ |
| D.4. Persevere to complete and master tasks. | ✓ | ✓ | ✓ |
| E.1. Work independently. | ✓ | ✓ | |
| E.2. Work collaboratively. | ✓ | ✓ | ✓ |
| F.1. Attribute ideas and information to source materials and people. | ✓ | ✓ | ✓ |
| F.2. Evaluate sources for quality of content, validity, credibility, and relevance. | | ✓ | |
| *II. Foundational Skills* | | | |
| B.1. Write clearly and coherently using standard writing conventions. | | ✓ | ✓ |
| C.5. Synthesize and organize information effectively. | ✓ | ✓ | ✓ |
| D.1. Identify patterns or departures form patterns among data. | | ✓ | ✓ |
| D.2. Use statistical and probabilistic skills necessary for planning and investigation and collecting, analyzing, and interpreting data. | ✓ | ✓ | ✓ |
| E.1. Use technology to gather information. | ✓ | ✓ | ✓ |
| E.2. Use technology to organize, manage, and analyze information. | ✓ | ✓ | ✓ |
| E.3. Use technology to communicate and display findings in a clear and coherent manner. | ✓ | ✓ | ✓ |

## TCCRS Mathematics Standards Addressed

| Performance Expectation | Getting Started | Investigating | Drawing Conclusions |
|---|---|---|---|
| *IV. Measurement Reasoning* | | | |
| B.1. Convert from one measurement system to another. | ✓ | ✓ | |
| B.2. Convert within a single measurement system. | | ✓ | |
| *VI. Statistical Reasoning* | | | |
| C.1. Make predictions and draw inferences using summary statistics. | | | ✓ |
| C.2. Analyze data sets using graphs and summary statistics. | | | ✓ |
| C.3. Analyze relationships between paired data using spreadsheets, graphing calculators, or statistical software. | | | ✓ |

## TEKS Standards Addressed

| ***Changing Effects -*** *Texas Essential Knowledge and Skills (TEKS): Math* |
|---|
| 111.34.b.8. Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to:<br>111.34.b.8.F. use conversions between measurement systems to solve problems in real-world situations. |
| 111.36.c.2. The student uses graphical and numerical techniques to study patterns and analyze data. The student is expected to:<br>111.36.c.2.A. interpret information from various graphs, including line graphs, bar graphs, circle graphs, histograms, scatterplots, line plots, stem and leaf plots, and box and whisker plots to draw conclusions from the data.<br>111.36.c.2.B. analyze numerical data using measures of central tendency, variability, and correlation in order to make inferences.<br>111.36.c.2.D. use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc., select the most appropriate model, and use the model to interpret information. |

# Changing Effects

## *Introduction*

Have you ever tried to speak to someone who did not speak your language? Or calculate the "dollar price" of an item in another currency? Civilizations across the world use an amazing variety of languages and monetary systems, but there are only two standardized measurement systems in common use around the world: metric and standard (or, "English system", from Great Britain's days as the world power). Depending on where you live in the world, one system of measurement is more prominent.

However, as we become more globalized, it is increasingly necessary to be able to translate accurately between measurement systems. How does this affect statistics gathered in parts of the world where different systems are used*?* What if a third, new system were to come into use?

## *Directions*

### Getting Started

1. Participate in a class discussion about the use of measurement systems in the world today.
2. Working in an assigned small group, develop a scenario in which a third system of measurement develops. Think about the kind of data (type of unit within a measurement system) that might be relevant to the scenario. Bear in mind that it must be comparable to either the standard or the metric system (as opposed to data reflecting money, age, or race). Name this new system and create a "new" unit for the measurement that is relevant to your scenario. What would be the conversion factor? Why?

### Investigating

1. Using your thinking about a type of data relevant to the scenario, locate an appropriate data set online. Be sure to cite the source of your data set.
2. Convert your data set (or a sample of your data set) to your invented system using the conversion factor you determined. Choose a way to represent the results so that someone new to your system can easily interpret them.

3. Compute measures of central tendency and spread for the converted data set. If your data set is univariate, also compute the *z*-score for one of your data points. If your data set is bivariate, also determine the least-squares regression line and correlation coefficient.
4. Make a conjecture about how your conversion affects each part of the statistical analysis you performed above. In each case, describe what you would need to do to present your results to someone using the metric or standard system. Record your thinking so it is clear to an outside reader.
5. Conduct the same statistical analyses on the original data set.

## Drawing Conclusions

1. Independent of your group, think about the following. Take notes on your thinking to use in writing your report.
   - What are the effects of changing measurement units on data and the statistics derived from that data?
   - What is the implication of this conclusion for science conducted globally?
   - Does the choice of units reveal anything that another choice would not? Does it enable easier calculations, or reveal more evident observations of fact or conclusion?
2. Write a report of your conclusions. Use examples from your group's work to support your conclusions and to clarify the points you wish to make. Include representations that help you communicate clearly to an outside reader.