Area Exploration

Overview

Description

In this activity, students will explore the computation of area by calculating the cost of pools per square foot. They will start by looking at simple designs that incorporate shapes for which they can easily calculate the area. Next, they will calculate the area of more complicated pool shapes. Students are expected to struggle with finding the area of these shapes. The instructor will guide them in finding ways to estimate the areas of these shapes. Finally, the instructor will introduce the idea of limits.

Final Product: Students should produce a report that includes a diagram of their pool, a description of how they estimated the area and calculated the cost, and a description of how the idea of a limit can help you find a close approximation of the area of curved shapes.

Subject

Pre-Calculus

Task Level

Grade 11

Objectives

Students will:

- Calculate the area of simple geometric shapes.
- Estimate the area of amorphous geometric shapes using rectangles.
- Understand the concepts of estimation, precision, and limit.

Preparation

- Make a copy of the Student Notes for each student. The students will need pencils, paper, and rulers.
- Plan the questions you will need to guide the students through Getting Started and Investigating

Prior Knowledge

- Students should understand and be able to calculate the area of simple geometric figures, specifically rectangles.

Key Concepts and Terms

- Area
- Estimation
- Limit
- Precision
- Square foot

Time Frame

The assignment can be done in about five 50-minute class periods. This assignment can be modified to meet the needs of different classroom schedules and student ability levels.

Instructional Plan

Getting Started

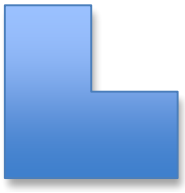
Learning Objectives

Students will:

- Review how to compute the area of a figure using basic geometric knowledge and skills.

Procedure

1. Introduce the problem to the students: Tell them they are going to be using their math skills to figure out the area and price of swimming pools.
2. Pass out the Student Notes. Tell them that we are going to be working through this assignment together for the next several days.
3. Ask the students to draw a rectangular swimming pool, label the dimensions of the swimming pool and calculate the area. Students may work individually, in pairs, or small groups.
4. Ask several students to share the dimensions and area of their pool. Ask the students how they calculated the area.
5. Show the students a drawing of an L-shaped pool. For example:



   Give the student dimensions for the pool and ask them to find the area.
6. Ask students to explain how they calculated the area.
7. Ask students to calculate the price of the pool if the price per square foot is $10.

Investigating

Learning Objectives

Students will:

- Estimate the area of amorphous shapes.
- Discuss precision in measurement.

- Understand the concept of limit.

Procedure

1. Show the students a drawing of a pool with curved sides. For example:

2. Ask students to find the area of this pool so they can calculate the price. Give them some time to think and discuss with their classmates. Circulate and monitor the discussion. Answer the students' questions and ask guiding questions.
3. Once most students have figured out that they can estimate the area of the pool by adding together the areas of rectangles that fit inside the pool, have students share their solutions with the class.
4. Ask the class if they see any problems with this solution. Prompt them (if needed) to notice the part of the swimming pool that is not being measured because the rectangles do not cover the entire area.
5. Point out that this is an estimation and not an actual measurement of the area of the pool. Discuss the concept of precision.
6. Ask the students how they could improve their measurement or, in other words, make it more precise. Let them think about this and discuss it with their classmates. Circulate and monitor the discussion. Answer the students' questions and ask guiding questions.
7. Guide the students to understand that the way to get a more precise measurement is to make the rectangles smaller and smaller.
8. Introduce the concept of limit as a tool mathematicians use to solve this problem.

Drawing Conclusions

Learning Objectives

Students will:

- Estimate the area of an amorphous shape.
- Calculate cost based on area.
- Communicate the mathematics they used to solve a problem.

Procedure

1. Have students individually prepare a report of a swimming pool they design. The pool should be a shape with curved sides. In their report there should be

a diagram of their pool and a description of the measurements they took and calculations they made to estimate the area and find the cost. Finally, they should describe how the idea of limit can help them find the best approximation of the area of their pool.

Scaffolding/Instructional Support

The goal of scaffolding is to provide support to encourage student success, independence, and self-management. Instructors can use these suggestions, in part or all together, to meet diverse student needs. The more skilled the student, however, the less scaffolding that he or she will need. Because of the extended nature of this activity, help might be needed at various stages. Below are questions that may be useful in guiding students through the assignment.

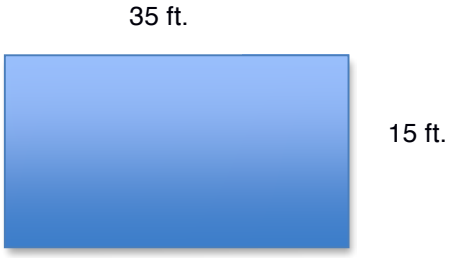
- Can you use a shape that you know how to calculate the area of to help you calculate the area of an amorphous shaped pool?
- Is this method of calculating the area completely accurate?
- Is there a way you could make the measurement more precise?
- Is there a way to eliminate the space that is not being covered by rectangles when you are trying to estimate the area?
- Is there always going to be part of the area of the pool that you are taking into account no matter how small you make the rectangles?

Solutions

The information below is intended to help you assess students' final work products. It may not represent all possible strategies and ideas. The accompanying scoring guide provides specific examples of ways a student might demonstrate content understanding and mastery of cross-disciplinary skills.

Getting Started:

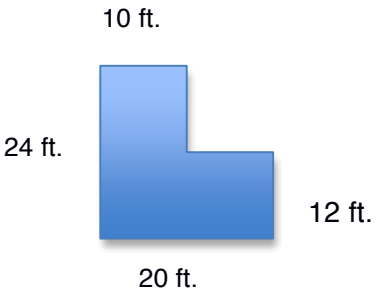
3. Student solutions will vary. A typical example is as follows:



Area = 35 x 15

= 525

5. Student solutions will vary. A typical example is as follows:

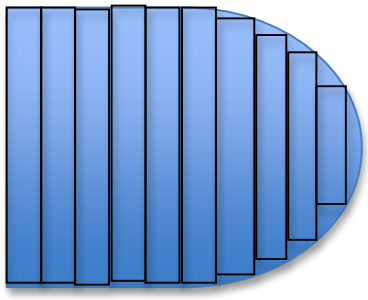


(10 x 12) + (12 x 20) = 360 sq. ft.

Price = $10 x 360 sq. ft. = $3,600

Investigating:

Students can measure their rectangles using rulers.



Drawing Conclusions:

1. Student answers will vary. Their report should include drawings with labels and an explanation of how they estimated the area. It should also include an explanation of how the idea of limit can help estimate the area of shapes with curved sides.

TCCRS Cross-Disciplinary Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Key Cognitive Skills			
A.1. Engage in scholarly inquiry and dialogue.	✓	✓	✓
A.2. Accept constructive criticism and revise personal views when valid evidence warrants.	✓	✓	✓
B.1. Consider arguments and conclusions of self and others.	✓	✓	✓
B.2. Construct well-reasoned arguments to explain phenomena, validate conjectures, or support positions.	✓	✓	✓
B.4.Support or modify claims based on the results of an inquiry.	✓	✓	✓
D.1. Self-monitor learning needs and seek assistance when needed.	✓	✓	✓
D.3. Strive for accuracy and precision.	✓	✓	✓
D.4. Persevere to complete and master tasks.	✓	✓	✓
E.2. Work collaboratively.	✓	✓	✓
F.2. Evaluate sources for quality of content, validity, credibility, and relevance.		✓	
II. Foundational Skills			
B.1. Write clearly and coherently using standard writing conventions.	✓	✓	✓

TCCRS Mathematics Standards Addressed

Performance Expectation	Getting Started	Investigating	Drawing Conclusions
I. Numeric Reasoning			
B.1. Perform computations with real and complex numbers.	✓	✓	✓
III. Geometric Reasoning			
C.3. Make connections between geometry and measurement.	✓	✓	✓

IV. Measurement Reasoning			
C.1. Find the perimeter and area of two-dimensional figures.	✓	✓	
VIII. Problem Solving and Reasoning			
A.1. Analyze given information.	✓	✓	✓
A.2. Formulate a plan or strategy.	✓	✓	✓
A.3. Determine a solution.	✓	✓	✓
A.4. Justify the solution.	✓	✓	✓
IX. Communication and Representation			
A.2. Use mathematical language to represent and communicate the mathematical concepts in a problem.	✓	✓	✓
C.1. Communicate mathematical ideas, reasoning, and their implications using symbols, diagrams, graphs, and words.	✓	✓	✓
C.3. Explain, display, or justify mathematical ideas and arguments using precise mathematical language in written or oral communications.	✓	✓	✓

TEKS Standards Addressed

Area Exploration *- Texas Essential Knowledge and Skills (TEKS): Math*
111.34.b.8. Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. The student is expected to: 111.34.b.8.A. find areas of regular polygons, circles, and composite figures;
111.34.b.9. Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures. The student is expected to: 111.34.b.9.B. formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models;
111.35.c.1. The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to: 111.35.c.1.E. investigate the concepts of continuity, end behavior, asymptotes, and limits and connect these characteristics to functions represented graphically and numerically.

Area Exploration

Introduction

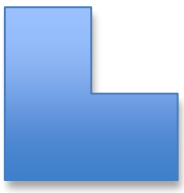
In this assignment you will be finding the area of swimming pools of different shapes. Once you find the area you will be able to calculate the cost.

Directions

Follow the directions below. You will discuss each of the steps below with your classmates. Your instructor will provide you with the dimensions of the pools.

Getting Started

1. On a piece of paper, draw a simple rectangular swimming pool. Label the dimensions of the pool and find the area.
2. Find the area of a pool with the following shape:



3. Calculate the cost of the pool if the cost is $10 per square ft.

Investigating

1. Find the area of a pool with curved sides.

Drawing Conclusions

1. Draw a pool of the shape of your choosing. Estimate the area of the pool and calculate the cost of the pool at $10 per square foot.
2. Write a report that describes how you estimated the area your pool and calculated the cost. Your report should include a diagram of your pool. Finally, describe how the idea of a limit can help you find the best possible approximation of the area.